Accred Qual Assur
DOI 10.1007/s00769-017-1291-7

GENERAL PAPER

Classification of objects into quality categories in the presence of hierarchical decision-making agents

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Received: 12 April 2017 / Accepted: 19 September 2017

Abstract In many practical contexts, it is often required to classify some objects of interest into predetermined unordered quality categories. This operation—referred to as *quality classification problem*—has received considerable attention in many fields of research, such as Analytical Chemistry, Materials Science, Medicine, Manufacturing, Quality Engineering/Management, Decision Analysis. Assuming that multiple agents perform *subjective* assignments of categories to the objects of interest, a further problem is that of fusing these assignments into global classifications. To this purpose, the *mode* and the *weighted mode* are very practical measures, as long as agents are equi-important or their (different) importance is expressed in the form of a set of weights. Unfortunately, these measures are not appropriate for quality classification problems where the agents' importance is expressed in the form of a rank ordering (hierarchy). The aim of this article is to present a new method, which addresses the latter quality classification problem in a relatively simple and practical way. The peculiarity of this method is that the different importance of agents determines a different priority in considering their assignments and not a different weight of these assignments. A detailed description of the new method is supported by a realistic example in the Analytical Chemistry field.

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Keywords Quality classification problem · Unordered quality categories · Nominal scale · Data fusion technique · Decision-making agents · Rank-ordered agents · Local-classification fusion

Introduction

According to the definition by Stevens [1], scales including unordered categories are defined as *nominal*. When using these scales, it is often required to classify an *object* of interest (o_1, o_2, o_3, etc.) into one of the scale categories (c_1, c_2, c_3, etc.). By an *object* we will consider a specific feature/attribute of the entity observed, e.g., a morphological characteristic of biological species (such as skin and eye color) or the sexual orientation of individuals (heterosexuality, homosexuality, bisexuality, asexuality, etc.).

This classification operation is referred to as *quality classification* or *nominal sorting* problem [2–6]. Quality classification can be *objective* or *subjective*, depending on the fact that objective and incontrovertible rules for driving it are available or not [7, 8]. For example, the classification of individuals according to their marital status (i.e., single, married, separated, divorced, widowed, civil union) is objective, while the classification of surface defects on hot-rolled steel plates by human visual inspection is subjective, as it may change from subject to subject. The classification problem has numerous practical applications in several fields, including but not limited to:

- *Analytical Chemistry/Materials Science*: classification of reference materials [9–12];
- *Medicine*: classification of patients into diseases groups [13, 14];

Published online: 07 October 2017

- *Biology*: classification of biological species based on their morphological characteristics [15];
- *Manufacturing:* online detection and classification of wafer bin map defect patterns [16]; multi-criteria ABC inventory classification with mixed quantitative and qualitative criteria [17];
- *Surface Defect Classification*: classification of defects on the surface of manufactured parts, by human visual inspection or machine vision [18, 19];
- *Quality Management*: customer satisfaction measurement of product/service features [20];
- *Engineering Design*: modeling Customer Requirements and Engineering Characteristics in QFD [21–23];
- *Human Resources Management*: assignment of personnel into appropriate occupation groups, according to their qualifications [24];
- *Pattern Recognition*: examination of the physical characteristics of objects, e.g., letter recognition [25];
- *Bibliometrics*: subject categorization of scientific journals [26].

In the scientific literature, the *quality classification problem* is addressed through a variety of models, which share the following features: (1) each quality category is defined a priori and characterized by one or more typical objects, also known as *reference objects* or *prototypes*; (2) a set of criteria are used for comparing the object of interest with the reference object(s) of each quality category; (3) the most plausible quality category for the object of interest is the one minimizing a suitable dissimilarity measure. For more information on the existing models, we refer the reader to the vast literature and extensive reviews [3, 27].

In this paper, we analyze the quality classification problem from a different perspective: we assume that M *decision-making agents* (d_1–d_M) should have to classify the object of interest into a plausible quality scale category. By a *decision-making agent*, we will consider any of a wide variety of different types of entities; examples could be human beings, individual criteria in a multi-criteria decision process, software-based intelligent agents on the Internet, etc.

Returning to the problem, (i) each agent performs a (subjective) *local* classification, selecting the most plausible quality category (e.g., agent d_2 classifies object o_1 into quality category c_3) and (ii) the agents' local quality classifications are fused into a *global* one. No matter how local classifications are produced (e.g., based on suitable models, agents' experience/intuition); what only matters is to find a method to fuse them.

The terminology in use is closely aligned with that of the "Vocabulary for nominal properties and nominal examinations" [28], which is aimed at scientists of disciplines in Clinical Laboratory Science, Analytical Chemistry, Material Science (see the comparison in Table 1).

Another definition of the problem of interest would be that in which agents formulate local preference orderings of the quality categories (e.g., $c_1 > c_2 \sim c_3 > \ldots$, where symbols ">" and "$\sim$", respectively, mean "strictly preferred to" and "indifferent to"), which are then aggregated into a unique consensus ordering. This other problem is very standard and has been discussed for over two centuries in a variety of real-life contexts, ranging from *multi-criteria decision aiding/making* to *social choice* theory [29]. However, selecting just the most plausible quality category is more practical for agents than formulating a preference ordering, for two main reasons:

- In general, the objective of the quality classification is to associate one-and-only-one nominal category with the object of interest and not to define a global ordering of the quality categories;
- In many practical situations, quality categories are mutually exclusive and/or inconsistent with each other, i.e., the selection of a category automatically excludes the others. For this reason, constructing local preference orderings can sometimes be unreasonable and complicated for agents. For example, considering the quality problem of classifying biological species (i.e., *bacteria*, *protozoa*, *chromista*, *plantae*, *fungi*, *animalia*), it seems unreasonable to construct orderings of the categories, if just one of them is likely to be appropriate, while the others not.

For the above reasons, in the remainder of this paper we will focus on a quality classification problem in which each agent has to select just the most plausible quality category.

Assuming that agents all have the same importance (*fully democratic* case), the easiest way to select the global quality category is through the *mode*, which is the quality category with the highest frequency of agents' assignments (e.g., considering the classification process exemplified in Table 2, the *mode* for object o_1 identifies category c_1). In some cases, the mode identifies multiple categories, with the same frequency (e.g., see the quality classification of object o_2, in Table 2).

Assuming that agents are not equi-important, the problem is a bit more complicated. The most common approach for representing the importance hierarchy of agents is through a set of weights (i.e., $w_1, w_2, \ldots, w_M$, defined on a cardinal scale), which reflect their recognized abilities and/or privileged positions of power [30]. When using weights, the global quality classification can be determined through the *weighted mode*, which identifies the category with the highest sum of agent weights (see the example in Table 3).

We remark that weight quantification is a very delicate task that can greatly affect the final result of the quality

Table 1 Comparison of several key terms/expressions in use with other equivalent ones, according to the "Vocabulary for nominal properties and nominal examinations" [28]

Terminology in use	Terminology of the "Vocabulary for nominal properties and nominal examinations"
Decision-making agent	Nominal examining system
Object	Object
Attribute	Nominal property/examinand
Quality category	Nominal property value
(Local or global) classification	Nominal examination result

Table 2 Classification of two fictitious objects (o_1 and o_2) into four nominal quality categories (c_1, c_2, c_3 and c_4), using the mode of the assignments by 5 equi-important agents (d_1–d_5)

Object	Agent assignments					Mode
	d_1	d_2	d_3	d_4	d_5	
o_1	c_1	c_2	c_1	c_1	c_3	c_1 (3 out of 5 total assignments)
o_2	c_3	c_4	c_4	c_3	c_1	c_3, c_4 (2 out of 5 total assignments)

classification. Although the literature provides several techniques for guiding it—for example, the AHP procedure [30, 31], or the method proposed in [32]—they are often neglected in practice, probably because of their complexity or the (strong) hypotheses behind their use. As a result, weights are not rarely assigned in arbitrary and questionable ways.

In some situations, this issue can be partially overcome by expressing the agents' importance in the form of a (linear) *rank ordering*—such as $d_1 > (d_2 \sim d_3) > \ldots > d_M$, where again symbols "$>$" and "$\sim$", respectively, depict the "strict preference" and "indifference" relationship—instead of a set of weights defined on a *cardinal* scale [33]. The formulation of such a rank ordering is simpler and more intuitive than that of a set of weights, especially when the agent importance prioritization is uncertain [34–36].

To better understand the quality classification problem with (hierarchical) rank-ordered agents, let us give a real-world example. In the *continuous improvement* division of a company, developing and manufacturing air-conditioning systems for motor vehicles, a number of experts (d_1, d_2, d_3, etc.), i.e., the agents of the problem, have to classify several textual customer complaints (i.e., the objects of the problem) into some predetermined (nominal) failure categories (e.g., c_1—non-uniform temperature in the passenger compartment, c_2—slow windscreen demisting, c_3—vibrations) [34]. For each customer complaint, experts perform a local (subjective) classification into one or more failure categories. Also, experts have different technical skills and competences, reflected by an importance rank ordering, such as $d_2 > (d_1 \sim d_3) > \ldots$. The goal is to fuse the experts' local quality classifications into global ones. The analysis results can be used for driving the improvement process of air-conditioning systems.

The fact of expressing the agents' importance through a rank ordering makes the problem original in the scientific literature. Another problem with rank-ordered agents, in which agents have to formulate preference orderings of the categories, was introduced by Yager [37] and subsequently developed by other authors [23, 34, 35, 38, 39].

The objective of this paper is to present a new method for addressing the quality classification problem. This method should also be adaptable to realistic contexts, in which (i) agents are unable to classify some of the objects of interest and/or (ii) they may formulate multiple assignments for the same object.

The remainder of this paper is organized in three sections. Section "Description of the method" describes the method from a conceptual viewpoint. Section "Application example" presents a realistic application example, in the field of Analytical Chemistry. The concluding section summarizes the original contributions of this research, highlighting its practical implications, limitations and suggestions for future research.

Description of the method

This section is organized in three subsections; sections "Intuitive perspective" and "Mathematical perspective", respectively, describe the proposed method from an intuitive and a mathematical perspective. Section "Concise

Table 3 Classification of two fictitious objects (o_1 and o_2) into four nominal quality categories (c_1, c_2, c_3 and c_4), using the weighted mode of the assignments by 5 agents (d_1–d_5), with different weights (w_1 to w_5)

Object	Agent assignments					Weighted mode
	d_1 ($w_1 = 40$ %)	d_2 ($w_2 = 35$ %)	d_3 ($w_3 = 10$ %)	d_4 ($w_4 = 10$ %)	d_5 ($w_5 = 5$ %)	
o_1	c_1	c_2	c_1	c_1	c_3	c_1 ($\sum w_i = 60$ %)
o_2	c_3	c_4	c_4	c_3	c_1	c_3 ($\sum w_i = 50$ %)

analysis of the properties of the method" contains a concise analysis of the method on the basis of some popular axioms borrowed from the *social choice theory*.

Intuitive perspective

It is assumed that a set of objects (o_1, o_2, o_3, etc.) should be classified into a set of (unordered) quality categories (c_1, c_2, c_3, etc.) of a nominal scale. For each object, the local classifications by M agents (d_1, d_2, …, d_M) should be fused into a global one. We remark that the classification of an object into one of the quality categories is solely founded on the analysis of that object (independently from the other ones).

The quality classification process is based on several steps. Referring to a generic object of interest, each agent performs a local classification, assigning a vote and a corresponding unitary score to the preferred quality category; in the case n quality categories are selected for the same object, the vote is split equally between them, and a score $1/n$ is assigned to each of these categories. It is assumed that agents may classify an object in multiple quality categories, not to force them to dubious classifications in the case of hesitation. However, since the classification of an object into a single quality category, rather than multiple ones, certainly denotes a higher confidence level of the agent, it seems reasonable that—in the case of multiple assignments—the vote is fractionalized. Table 4 summarizes the proposed scoring system. If an agent is unable to classify the object of interest (e.g., when it is not well known), that agent will be excluded from voting. Among the possible scoring systems, the proposed one seems to be relatively natural and intuitive.

Next, for a generic object of interest, the following decision parameters are determined:

- m the number of voting agents, excluding those unable to classify the object. Obviously, m will coincide with the total score (given by the totality of the voting agents) and it will be smaller than or equal to M (i.e., $m \leq M$).
- q a *quorum* threshold. If $m \geq q$, the fusion process is performed as explained later in this section; if $m < q$, the fusion process is aborted, not resulting in any global classification (i.e., the result of the quality classification is *indeterminate*).
- t a *consensus* threshold used in the fusion process; later in this section it is explained how to determine suitable values of t.

The use of q is for preventing dubious global classifications, in "uncertain" situations where several agents are unable to classify the object. In other words, it seems reasonable not to force any global classification, when a relatively large portion of the agents find it difficult to perform their local classifications. As a first attempt, we conventionally set:

$$q = M/2, \tag{1}$$

although we are aware that this threshold may be varied depending on the required "degree of prudence".

In the case $m \geq q$, i.e., when the quorum threshold is reached, the global classification is determined as follows. For each object, the agents' votes are examined in several turns, proceeding in descending order with respect to their relative importance. If two (or more) agents have indifferent importance, their votes are examined in the same turn; for example, the agents' importance rank ordering $(d_1 \sim d_7) > (d_2 \sim d_4) > (d_5 \sim d_{10} \sim d_3) > (d_8 \sim d_9 \sim d_6)$ produces four turns, which include four groups of agents with mutual relationships of indifference—i.e., turn 1 includes d_1 and d_7, turn 2 includes d_2 and d_4, turn 3 includes d_5, d_{10} and d_3, and turn 4 includes d_8, d_9 and d_6. Of course, the total number of turns will be smaller than or equal to the number (m) of voting agents.

Table 4 Scores assigned to the quality categories, in the (local) classification of an object by a single agent

Case	Score assigned to	
	(i) Preferred quality category/ies	(ii) Other quality categories
Agent classifies the object into one-and-only-one category	1	0
Agent classifies the object into (n) multiple categories	$1/n$	0
Agent unable to classify the object	N/A	N/A

Table 5 Fictitious quality classification of an object of interest by four rank-ordered agents (d_1–d_4)

Agents	d_1	d_2	d_3	d_4
Assignments	c_1	c_2	c_2	c_1
Importance rank ordering $(d_1 \sim d_3) > d_4 > d_2$				
Consensus threshold $t = 2$				

	Turn 1 $d_1 \sim d_3$	Turn 2 d_4	Turn 3 d_2
Assignments	c_1, c_2	c_1	c_2
CUM(c_1)	1	2	2
CUM(c_2)	1	1	2

CUM(c_1) and CUM(c_2) are the cumulative scores of c_1 and c_2 categories respectively

Simplifying, the fusion process establishes that the first quality category whose cumulative score ("CUM") reaches a consensus threshold t (that we will focus on later) is the preferred one. For the purpose of example, Table 5 reports a simplified quality classification problem in which four agents (d_1–d_4) should identify the most plausible (nominal) quality category (c_1 and c_2) for an object of interest. Since the agents' importance rank ordering is $(d_1 \sim d_3) > d_4 > d_2$, their votes are examined in three turns. The resulting selected category is c_1, as it is the one whose cumulative score—i.e., CUM(c_1)—reaches t first (in turn 2).

The following pseudo-code illustrates the fusion of the agents' local quality classifications into a global one, for a generic object:

More precisely, there are three possible results of the global classification:

1. A *single quality category* This result may occur in two different cases: (i) when one (and only one) quality category reaches/exceeds t in one turn and (ii) when two or more quality categories reach/exceed t at the same turn and only one of them has a cumulative score ("CUM") higher than the others till that turn.
2. *Multiple categories* This result may occur when two or more quality categories reach/exceed t at the same turn, with the same cumulative score ("CUM") till that turn.
3. *Indeterminate* ("*Indet.*") This result may occur in two different cases: (i) when $m < q$ (i.e., the number of agents able to classify the object of interest does not reach the quorum threshold) and (ii) when $m \geq q$ but

```
1.  Define m as the number of agents able to classify the object of interest.
2.  Set the value of the quorum threshold q.
3.  Initialise the set of the "potentially selectable" quality categories, i.e., S, to Ø (empty).
4.  Initialise the cumulative score of each j-th quality category, i.e., CUM(j-th category), to 0.
5.  If m < q (quorum is not reached), then:
6.      The result of the quality classification is indeterminate.
7.  Else If m ≥ q (quorum is reached), then:
8.      Set the value of the consensus threshold t.
9.      Determine the number of turns.
10.     For each (i-th) turn:
11.         For each (j-th) quality category:
12.             CUM(j-th category) = CUM(j-th category) + the score received in that turn.
13.             If CUM(j-th category) ≥ t, then:
14.                 Include the j-th category in S.
15.             End If.
16.         End For.
17.         If S ≠ Ø, then:
18.             Select the global quality category(ies) ∈ S, with the maximum cumulative score, i.e.,
                CUM(j-th category) = Max_{for each jth category ∈ S} {CUM(j-th category)}.
19.             Exit For (Go To 22).
20.         End If.
21.     End For.
22.     If S = Ø, then:
23.         The result of the quality classification is indeterminate.
24.     End If.
25. End If.
26. End.
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the cumulative score ("CUM") of none of the quality categories' reaches t, in the totality of the turns. The latter situation can occur when using relatively high values of t.

The proposed method can be slightly adjusted, so as to "force" the selection of a single quality category, even in the situations described at point (2) or (3). For example, in the case (2) one could isolate the quality category that obtained the assignment of the most important agent among those able to classify the object of interest. On the other hand, in the case (3) one could select the category corresponding to the mode (i.e., the one with the highest frequency of assignments) or the one that obtained the assignment of the most important agent among those able to classify the object of interest. However, we remark that the introduction of these adjustments can be risky, because it may force to questionable global quality classifications, in intrinsically uncertain situations.

Returning to the selection mechanism that characterizes the proposed fusion method, it is worth noting that the different "voting power" of agents determines a different priority order when expressing their (unitary) vote. This is the most important difference with respect to other approaches of *social choice theory*, in which the agent vote can be weighted and/or there is no priority order when voting [30, 40–42]. Similar approaches have been used in [34] to solve the problem of fusing multi-agent preference orderings into a consensus ordering. In particular, the proposed method can be seen as a *top-down* variant of the so-called generalized Yager's algorithm (GYA) [23].

Let us now focus the attention on the rationale for choosing a suitable t value. As a first consideration, when $m/2 < t \leq m$, the global quality classification is not affected by the agents' voting order. In other words, when $m/2 < t \leq m$, the criterion for selecting one category over another degenerates into that of the majority, regardless of the agents' voting order. For instance, if c_1 has a total score larger than or equal to t, the global classification will certainly result in c_1, since there will be no voting sequence for which other categories can reach t before c_1. Let us consider the example in Table 6, in which the local classifications by four agents should be fused into a global quality classification. For simplicity, the (nominal) scale quality categories are just two: c_1 and c_2. Four among the possible agents' rank orderings are considered. When $m/2 = 2 < t \leq m = 4$, the global quality classification does not depend on the voting sequence (it is always indeterminate in this case); we provide a formal proof of this point in section "Further considerations on the range of no-voting-order-effect" (in the "Appendix"). For this reason, $]\,m/2, m]$ can be classified as range of *no-voting-order-effect* (see Fig. 1).

We remark that, as t increases within this range, the required level of agreement between agents increases. The extreme case is $t = m$, corresponding to a *unanimity* situation where the global classification will always be indeterminate, except when all the m agents involved share the same local classification.

On the other hand, in the case $1 \leq t \leq m/2$, the voting order may affect the global quality classification. For the purpose of example, Table 6 shows that, when $t = 1$ or $1 < t \leq m/2$, results may change depending on the voting order. Since the agents' voting order may affect the result of the fusion, the range $[1, m/2]$ may be denominated as range of *voting-order-effect*. There are two extreme cases: (i) $t = 1$, which denotes the *dictatorship* situation, in which the global quality classification coincides with that of the most important agent (able to classify the object of interest) and (ii) $t = m/2$, which denotes the borderline situation with respect to the *no-voting-order-effect* range.

Based on the previous considerations, a reasonable value of t can be that in the middle of the *voting-order-effect* range (see the representation in Fig. 1, where it is denoted by a star), i.e.:

$$t^* = \frac{1 + m/2}{2}. \tag{2}$$

We remark that the t^* value is purely conventional and a different value—as long as included in the *voting-order-effect* range—could lead to slightly different global quality classifications.

Mathematical perspective

This section describes the proposed method from a mathematical perspective. For practical reasons, the notation introduced in section "Intuitive perspective" will be slightly modified.

Consistently with the fact that one agent can classify the same object into multiple quality categories, let $C_o(d)$ be the set of quality categories into which agent d has classified object o. This defines a set $D_o = \{d \mid C_o(d)$ is not empty$\}$ of (m) agents that have provided a classification of o into at least one category and a local score $s_o(c, d)$ for each agent d in D_o, which is:

$$s_o(c, d) = \frac{1}{|C_o(d)|}, \tag{3}$$

where operator "| |" denotes the cardinality of the set of interest, i.e., the number of quality categories selected by agent d for object o. The cumulative score of an agent with respect to a quality category c is the sum of the scores of the agents (d^*) that (1) are more important than d or (2) have the same importance as d:

Table 6 Example of fusion of the local classifications by four agents into a global quality classification

Agents	d_1	d_2	d_3	d_4
Assignments	c_1	c_2	c_2	c_1
Score of c_1	1	0	0	1
Score of c_2	0	1	1	0

Agents' rank orderings	Global classification		
	$t = 1$	$1 < t \leq m/2$	$m/2 < t \leq m$
1: $d_1 > d_2 > d_3 > d_4$	c_1	c_2	N/A
2: $d_3 > d_2 > d_1 > d_4$	c_2	c_2	N/A
3: $(d_1 \sim d_3) > d_4 > d_2$	c_1, c_2	c_1	N/A
4: $d_1 \sim d_2 \sim d_3 \sim d_4$	c_1, c_2	c_1, c_2	N/A
…	?	?	N/A

In this case, $m = 4$

The result is calculated for different agents' rank orderings and t values

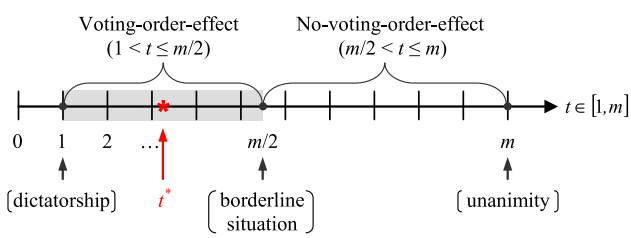

Fig. 1 Schematic subdivision of the range of variability of the t consensus threshold. The parameter m depicts the number of agents involved in the quality classification process

$$\mathrm{CUM}_o(c, d) = \sum_{d^* \geq d} s_o(c, d^*) \tag{4}$$

where symbol "$\geq$" denotes the strict preference or indifference relationship.

If D_o contains at least q agents, the global classification G_o of an object o is a set containing all quality categories c such that there exists an agent d having a cumulative score $\mathrm{CUM}_o(c, d)$ greater than or equal to the threshold t and there is no quality category $c^*$and no agent d^* such that $d^* \geq d$, $\mathrm{CUM}_o(c^*, d^*) \geq t$ and $\mathrm{CUM}_o(c^*, d^*) > \mathrm{CUM}_o(c, d^*)$. Otherwise, if D_o contains less than q agents, the global classification G_o is empty.

As a curiosity, it is simple to show that, in the case agents are equi-important, the proposed method degenerates to the *mode* method. In fact, since the $\mathrm{CUM}_o(c, d)$ values of a generic category c would be equal for all agents, the category in G_o would be the one exceeding t with the highest cumulative score, i.e., the one corresponding to the mode [43].

Concise analysis of the properties of the method

Table 7 presents a concise analysis of the proposed method from the viewpoint of some popular axioms borrowed from the *social choice* theory [44]. Since these axioms are generally used for the problem of aggregating preference orderings by multiple equi-important agents into a unique consensus ordering [45–47], we have taken the liberty to adapt them to the quality classification problem of interest. For example, we have replaced the expression *preference ordering* with *local quality classification*, *fused ordering* with *global quality classification*, *alternatives* with *quality categories*, etc. The (non-)fulfillment of these axioms can be demonstrated imitating the proofs of relevant theorems of social choice theory [44]. For example, the *idempotency* axiom is fulfilled since, assuming that the (m) agents classify the object of interest in the same quality category, at the end of the turns the cumulative score of this category will surely be $m \geq t$, while the cumulative score of the remaining categories will be zero. In a similar way, we can proceed for the remaining axioms.

Table 7 shows that the proposed method satisfies all the axioms except that of *anonymity*, which is obviously incompatible with the problem of interest, and that of *independence of irrelevant alternatives* (see the formal proof in section "Further considerations on the axiom of independence of irrelevant alternatives", in the "Appendix"). However, [48] shows that the negative consequences of the latter feature are not crucial.

Application example

This section presents an application example of the proposed method in the field of Analytical Chemistry, concerning the improvement of the clinical laboratory service in a hospital.

Through a market survey, a sample of ten respondents (d_1–d_{10})—i.e., physicians—were encouraged to describe the attributes (or customer requirements) of the service of interest. The list of attributes gathered in this exercise were refined and structured by a cross-functional team of experts (i.e., including clinical laboratory managers, quality managers, technicians and other hospital staff). The resulting 12 attributes in Table 8 were identified to represent the major concerns of physicians; for more information about these attributes, see [49].

Respondents were divided into four classes of importance (i.e., I, II, III and IV, in decreasing order), based on two analysis dimensions: (i) the "average frequency of test request" (e.g., a test required every x hours) and (ii) the "level of education of the respondent" (e.g., bachelor, master, doctorate). The team of experts selected these two

Table 7 Concise analysis of the proposed method, on the basis of some popular axioms adapted from *social choice theory*

Axiom	Description	
Idempotency	If all of the local classifications are the same, the resulting global quality classification is this one	✔
Anonymity	The system gives equal weight to each agent since, by permuting the agents' local classifications, the global quality classification does not change	✗[a]
Monotonicity	If any agent modifies his/her local classification by selecting a new quality category, then the global classification responds only by selecting the same quality category or not changing the initial one, never by selecting a third quality category different from the initial and the new one	✔
Non-dictatorship	The method accounts for the wishes of multiple agents. It cannot simply mimic the local quality classification of a single agent (dictator)	✔[b]
Unrestricted domain or *universality*	For any set of individual agent local classification, the algorithm yields a unique and global classification (no randomness)	✔
Independence of irrelevant alternatives	If a "non-global" quality category (i.e., a category different from the one(s) resulting from the global classification) is removed, then the global category(ies) remains unchanged	✗
Non-imposition or *citizen sovereignty*	Every selection of the quality category(ies) is possible as outcome	✔

The symbols "✔" and "✗", respectively, indicate the axioms satisfied or not

[a]The agents' rank ordering is not taken into account when $m/2 > t \geq m$ (see section "Intuitive perspective")

[b]A sort of condition of dictatorship would occur when $t = 1$ (see section "Intuitive perspective")

dimensions, as they may significantly influence the accuracy of the response while being relatively easy to evaluate. The two dimensions can be described through the two-dimensional map in Fig. 2. The most important respondents (in class I) are those with relatively high values in both the analysis dimensions. According to a lexicographic ordering, which favors the former dimension with respect to the latter, the second and third most important classes are, respectively, II and III. The least important respondents (in class IV) are those with relatively low values in both the analysis dimensions. Of course, the importance ranking could be based on additional and/or substitute analysis dimensions (e.g., "age of the respondents", "average complexity of requested tests") or different evaluation criteria.

Based on the above considerations, the resulting importance rank ordering of respondents is: $(d_1 \sim d_7) > (d_2 \sim d_4) > (d_5 \sim d_{10} \sim d_3) > (d_8 \sim d_9 \sim d_6)$.

Next, the ten respondents (i.e., the agents of the problem) should classify the 12 attributes in Table 7 (i.e., the objects of the problem), into the five (nominal) quality categories [i.e., *basic* or *must-be* (*B*), *one dimensional* (*O*), *excitement* (*E*), *indifferent* (*I*) and *reverse* (*R*)] according to the Kano model [50]; Table 9 provides a synthetic description of these quality categories. The Kano categorization may contribute to measure customer satisfaction and arrive at decisions regarding the introduction of new features, or extension/enhancement of some features for the clinical laboratory service to be improved [51].

The Kano categories (*B*, *O*, *E*, *I* and *R*) are *subjective*, as their assignment to the object of interest may change from respondent to respondent. Moreover, these quality categories are defined on a *nominal* scale, as there are not indisputable ordering relations between the various categories; e.g., one cannot say whether category *B* is more important than category *O* or *E*.

The importance class of each respondent and the relevant local classifications are reported in Table 10. It can be noticed that respondents are often unable to classify some of the attributes (see the last row).

It may be noted that for only five attributes (i.e., o_1, o_6, o_7, o_8 and o_{11}), the totality of the agents perform a local quality classification of the object of interest; for the other seven attributes, there is at least one respondent unable to perform the quality classification. We also observe that for 23 out of 99 (effective) assignments, respondents select multiple categories, denoting some hesitation.

Table 11 shows the fusion of the respondents' local classifications into corresponding global quality classifications. For each object, the decision parameters q and t are determined by applying Eqs. 1 and 2, respectively; the resulting quality classifications are reported in the penultimate column. Given that there are four groups of respondents with mutual relationship, there will be four total turns in which agents' votes are examined.

We note that the solutions generated by the proposed method do not necessarily coincide with those obtained through the mode (in the last column of Table 11); the reason is that the mode neglects the agents' importance hierarchy. Not surprisingly, when all agents are equi-important (fully democratic case), the results of the proposed method "degenerate" into those obtained through the mode.

Table 8 List of the major attributes related to a hospital clinical laboratory, from the perspective of physicians

Abbr.	Description
o_1	Good quality/reliability of results
o_2	Reasonable turnaround time of routine tests
o_3	Reasonable turnaround time of urgent tests
o_4	Adequate test menu
o_5	Accessibility of laboratory manager
o_6	Accessibility of laboratory pathologists
o_7	Adequate notification of critical values
o_8	Clear format of the clinical report
o_9	Accessibility of laboratory staff
o_{10}	Reasonable turnaround time of esoteric tests
o_{11}	Courteous staff
o_{12}	Responsive laboratory management

Let us now focus on the construction of the global quality classification, for some of the attributes examined: o_3, o_5, o_8 and o_{11}. Regarding o_3, the result is indeterminate because the number of respondents able to classify that object is too small (i.e., $m = 4 \leq q = 5$). Regarding o_5, the global classification results in quality category O, since CUM(O) reaches t first (in turn 3). Regarding o_8, the global classification is indeterminate, since the cumulative score for any of the five quality categories fails to reach t (which is equal to 3), in all the four turns. Regarding o_{11}, the global classification results in two tied quality categories (i.e., B and E), as their relevant cumulative scores reach (and overcome) t at the same turn (i.e., turn 3). Figure 3 shows the step-by-step construction of the global quality classification for these four attributes.

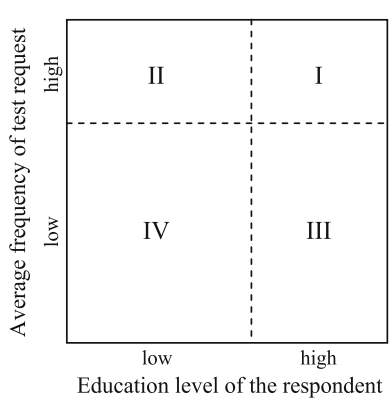

Fig. 2 Qualitative map to discriminate the importance classes (I, II, III and IV) of respondents, according to two dimensions of analysis (i.e., "average frequency of test request" and "education level of the respondent")

Table 9 Short description of the Kano quality categories [51]

	Category	Description
B	Basic or must-be	These attributes are taken for granted when fulfilled, but result in dissatisfaction when not fulfilled, e.g., an ATM service that dispenses the correct amount of cash, without running out of money
O	One dimensional	These attributes result in satisfaction when fulfilled and dissatisfaction when not fulfilled, e.g., music system and air-conditioning facility in cars
E	Excitement	These attributes provide satisfaction when fully achieved, but do not cause dissatisfaction when not fulfilled. These are attributes that are not normally expected, e.g., a battery with an integrated power-check mechanism
I	Indifferent	These attributes refer to aspects that are neither good nor bad, and they do not result in either customer satisfaction or customer dissatisfaction, e.g., the style of inscription of company name on soap
R	Reverse	These attributes refer to a high degree of achievement resulting in dissatisfaction and to the fact that not all customers are alike, e.g., some customers prefer high-tech products, while others prefer the basic model of a product and will be dissatisfied if a product has too many extra features

Discussion

The proposed method makes it possible to classify objects into predetermined quality categories of a nominal scale, in the specific case in which (i) multiple agents formulate their subjective assignments about the most plausible quality category(ies) and (ii) the agents' importance is expressed in the form of a rank ordering (not the "canonical" set of weights). The method can be applied to a variety of situations, in which objective/quantitative rules for guiding the quality classification process are not available.

The most original aspect of the proposed method is that the different "voting power" of agents determines a different voting priority, and not a different weight of the vote; the choice of the t value makes it possible to switch with a continuity from the situation of *dictatorship* (when $t = 1$) to that of *democracy* (when $t > m/2$).

The proposed method is simple, intuitive and automatable; it is also rather flexible, as it can be applied in situations of hesitation, where some agents perform multiple assignments or are unable to classify the object of interest.

A potential limitation of the proposed method is that it requires the arbitrary quantification of the (q and t) threshold values. Also, it does not always lead to a global quality

Table 10 Local quality classifications related to the ten respondents surveyed (d_1–d_{10})

Respondents	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
Importance class	I	II	III	II	III	IV	I	IV	IV	III
Local classification										
o_1	*O*	*B*	*O*	*B*, *E*	*I*	*B*, *O*	*B*	*E*	*B*	*E*
o_2	*O*	*O*, *I*	*B*	–	*O*	*O*	*B*	*O*	*E*	*I*
o_3	–	–	*B*, *O*	*I*	–	–	*B*	–	*O*	–
o_4	*B*, *O*	–	*B*	*B*	–	*O*	*O*	–	*B*, *O*	–
o_5	*B*	*O*	*I*, *O*	–	*I*, *E*, *O*	*I*	*O*	*I*	*B*	*I*
o_6	*B*, *E*	*I*	*B*, *I*	*B*	*I*	*I*	*B*	*I*	*O*	*B*, *I*
o_7	*E*	*O*	*E*	*O*, *E*, *I*	*B*	*I*	*O*	*B*	*E*	*O*
o_8	*O*	*E*	*E*	*R*	*R*, *O*	*I*	*O*	*R*	*E*, *I*	*I*
o_9	*E*	*B*, *E*	–	*B*, *O*, *I*	–	–	–	–	*B*, *E*	*E*, *I*
o_{10}	*B*	*I*	*I*	*I*	*B*, *I*	–	*O*	*E*	*I*	–
o_{11}	*E*	*B*	*B*, *E*	*E*	*B*	*E*	*B*	*E*	*R*, *E*	*E*
o_{12}	*O*	–	*I*	–	*O*	*I*	*O*, *I*	*I*	*E*	*O*, *O*, *I*
Unclassified attributes	$\{o_3\}$	$\{o_3, o_4, o_{12}\}$	$\{o_9\}$	$\{o_2, o_5, o_{12}\}$	$\{o_3, o_4, o_9\}$	$\{o_3, o_9, o_{10}\}$	$\{o_9\}$	$\{o_3, o_4, o_9\}$	Ø	$\{o_3, o_4, o_{10}\}$

classification of the object of interest, but it can lead to multiple or indeterminate classifications. This apparent limitation represents a form of protection from determining controversial quality classifications, in situations of great hesitation (e.g., when several agents are not able to classify the object of interest and/or their local classifications are very discrepant).

Future research go in two directions: (i) sensitivity analysis of the robustness of the proposed method with respect to small variations in the local quality classifications and/or in the q and t thresholds, (ii) comparison with other methods based on the degree of consistency between the global classification and the input data [35] and (iii) application of the method to other practical cases, in various scientific fields.

Appendix

Further considerations on the range of no-voting-order-effect

Let us provide a formal proof that when $t \in]m/2, m]$ (i.e., t is included in the no-voting-order-effect range), the global quality classification does not depend on the voting sequence.

For a single object of interest, given that (1) the sum of the scores (s) assigned by agents (d) to the quality categories (c) is equal to m (see weight definition in Table 4), i.e.,

$$\sum_d \sum_c s(c, d) = m, \tag{5}$$

and (2) $m/2 < t \leq m$, if there exists a quality category (c^*) whose total score (obtained cumulating the scores assigned by the m agents) exceeds t, i.e.,

$$\text{one element } c^* \text{ exists} \left| \frac{m}{2} < t \leq \sum_d s(c^*, d) \leq m, \right. \tag{6}$$

then c^* will be the only global quality category, since the total score of no other category will be able to reach t, no matter the agents' rank ordering.

Assuming *ad absurdum* that there exists a second global quality category ($c^\dagger$), then we would have that

$$\sum_d s(c^*, d) \geq t > \frac{m}{2} \quad \text{and} \quad \sum_d s\left(c^\dagger, d\right) \geq t > \frac{m}{2}, \tag{7}$$

since the total scores of c^* and $c^\dagger$ would be both supposed to reach t. The sum of the total scores of the two quality categories c^* and $c^\dagger$ (neglecting the remaining ones) would therefore be

$$\sum_d s(c^*, d) + \sum_d s\left(c^\dagger, d\right) \geq 2 \cdot t > m, \tag{8}$$

which is absurd, because it is incompatible with the condition in Eq. 5.

This demonstrates that if $t \in]m/2, m]$ and a quality category c^* has a total score exceeding t, this category will be the only global quality category, independently on the agents' rank ordering.

Further considerations on the axiom of independence of irrelevant alternatives

This section provides a proof that the proposed method does not satisfy the axiom of independence of irrelevant alternatives (IIA).

Consider a quality classification problem where an object has been evaluated by three agents ($d_1 > d_2 > d_3$) and classified into 4 quality categories (c_1–c_4) as follows:

$d_1 : c_3, c_4;$

$d_2 : c_1;$

$d_3 : c_1, c_2, c_3, c_4.$

In this case $M = m = 3$, so let us assume $t = 1.25$ and $q = 1.5$. Then, according to the proposed procedure, category c_1 is selected at turn 3 with a cumulative score of 1.25. Let us comment this result. We notice that c_1 is selected by d_2 and d_3, while quality categories c_3 and c_4 are selected by d_1 and d_3, but overall they are not selected

(i) Classification of object o_3

Turns:	Turn 1		Turn 2		Turn 3			Turn 4			Parameters		
Agents:	d_1	d_7	d_2	d_4	d_3	d_5	d_{10}	d_6	d_8	d_9	m	q	t
Local classifications (scores):	- (0)	B (1)	- (0)	I (0)	B, O (0.5)	- (0)	- (0)	- (0)	- (0)	O (0)	4	5	1.5
Global classification is indeterminate, since $m < q$.													

(ii) Classification of object o_5

Turns:	Turn 1		Turn 2		Turn 3			Turn 4			Parameters		
Agents:	d_1	d_7	d_2	d_4	d_3	d_5	d_{10}	d_6	d_8	d_9	m	q	t
Local classifications (scores):	B (1)	O (1)	O (1)	- (0)	I, O (0.5)	I, E, O (0.33)	I (1)	I (1)	I (1)	B (1)	9	5	2.75
Cumulative scores: CUM(B)	1		1		1			2					
CUM(O)	1		2		2.83			2.83					
CUM(E)	0		0		0.33			0.33					
CUM(I)	0		0		1.83			3.83					
CUM(R)	0		0		0			0					
Set of selectable categories (S)	Ø		Ø		$\{O\}$			$\{O, I\}$					
Classification results in the selection of category O, since CUM(O) reaches t first (in turn 3).													

(iii) Classification of object o_8

Turns:	Turn 1		Turn 2		Turn 3			Turn 4			Parameters		
Agents:	d_1	d_7	d_2	d_4	d_3	d_5	d_{10}	d_6	d_8	d_9	m	q	t
Local classifications (scores):	O (1)	O (1)	E (1)	R (1)	E (1)	R, O (0.5)	I (1)	I (1)	R (1)	E, I (0.5)	10	5	3
Cumulative scores: CUM(B)	0		0		0			0					
CUM(O)	2		2		2.5			2.5					
CUM(E)	0		1		2			2.5					
CUM(I)	0		0		1			2.5					
CUM(R)	0		1		1.5			2.5					
Set of selectable categories (S)	Ø		Ø		Ø			Ø					
Global classification is indeterminate, since no category reaches t.													

(iv) Classification of object o_{11}

Turns:	Turn 1		Turn 2		Turn 3			Turn 4			Parameters		
Agents:	d_1	d_7	d_2	d_4	d_3	d_5	d_{10}	d_6	d_8	d_9	m	q	t
Local classifications (scores):	E (1)	B (1)	B (1)	E (1)	B, E (0.5)	B (1)	E (1)	E (1)	E (1)	R, E (0.5)	10	5	3
Cumulative scores: CUM(B)	1		2		3.5			3.5					
CUM(O)	0		0		0			0					
CUM(E)	1		2		3.5			6					
CUM(I)	0		0		0			0					
CUM(R)	0		0		0			0.5					
Set of selectable categories (S)	Ø		Ø		$\{B, E\}$			$\{B, E\}$					
Classification results in the selection of categories B and E, since CUM(B) and CUM(E) both reach t in turn 3, with the same score.													

Fig. 3 Step-by-step construction of the global quality classifications for four attributes (i.e., o_3, o_5, o_8 and o_{11})

Table 11 Fusion of the respondents' local classifications in Table 10 into corresponding global quality classifications

Attrib.	Turn 1		Turn 2		Turn 3			Turn 4			Parameters			Global classific.	Mode
	d_1	d_7	d_2	d_4	d_3	d_5	d_{10}	d_6	d_8	d_9	m	q	t		
o_1	*O*	*B*	*B*	*B*, *E*	*O*	*I*	*E*	*B*, *O*	*E*	*B*	10	5	3	*B*	*B*
o_2	*O*	*B*	*O*, *I*	–	*B*	*O*	*I*	*O*	*O*	*E*	9	5	2.75	*O*	*O*
o_3	–	*B*	–	*I*	*B*, *O*	–	–	–	–	*O*	4	5	1.5	Indet.	*B*, *O*
o_4	*B*, *O*	*O*	–	*B*	*B*	–	–	*O*	–	*B*, *O*	6	5	2	*B*	*B*, *O*
o_5	*B*	*O*	*O*	–	*I*, *O*	*I*, *E*, *O*	*I*	*I*	*I*	*B*	9	5	2.75	*O*	*I*
o_6	*B*, *E*	*B*	*I*	*B*	*B*, *I*	*I*	*B*, *I*	*I*	*I*	*O*	10	5	3	*B*	*I*
o_7	*E*	*O*	*O*	*O*, *E*, *I*	*E*	*B*	*O*	*I*	*B*	*E*	10	5	3	*O*	*O*, *E*
o_8	*O*	*O*	*E*	*R*	*E*	*R*, *O*	*I*	*I*	*R*	*E*, *I*	10	5	3	Indet.	*O*, *E*, *I*, *R*
o_9	*E*	–	*B*, *E*	*B*, *O*, *I*	–	–	*E*, *I*	–	–	*B*, *E*	5	5	1.75	*E*	*E*
o_{10}	*B*	*O*	*I*	*I*	*I*	*B*, *I*	–	–	*E*	*I*	8	5	2.5	*I*	*I*
o_{11}	*E*	*B*	*B*	*E*	*B*, *E*	*B*	*E*	*E*	*E*	*R*, *E*	10	5	3	*B*, *E*	*E*
o_{12}	*O*	*O*, *I*	–	–	*I*	*O*	*O*, *O*, *I*	*I*	*I*	*E*	8	5	2.5	*O*	*I*

The agents' importance rank ordering is: $(d_1 \sim d_7) > (d_2 \sim d_4) > (d_3 \sim d_5 \sim d_{10}) > (d_6 \sim d_8 \sim d_9)$, which determines four groups of mutually indifferent agents (and four relevant turns)

m, q and t are, respectively, the number of voting respondents, the quorum threshold and the consensus threshold relating to the object of interest; for each object, q and t values are determined by applying Eqs. 1 and 2, respectively

S is the set of quality categories reaching/overcoming t in a certain turn

(their cumulative scores are both 0.75), although they are chosen by a more important agent (d_1). The reason is that the assignments by d_1 and d_3 are both multiple and inherently uncertain, and consequently the relevant scores are fractionalized; on the other hand, the single assignment by d_2 entails a full score to c_1.

Now assume that quality category c_4 is removed from the analysis:

$$d_1 : c_3;$$
$$d_2 : c_1;$$
$$d_3 : c_1, c_2, c_3.$$

In this case, c_1 and c_3 are both selected, each having a score of 1.33 at turn 3. It is clear that the removal of the "non-global" quality category c_4 changed the result, thus leading to a violation of the IIA axiom. It can be shown that, in the particular case in which agents classify the object into a single quality category (i.e., no multiple assignments), the IIA axiom is fulfilled.

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